TRINITY COLLEGE

Year 12 Mathematics Methods Unit 3 & 4
Test 1 2021

Section 1 Calculator Free

Applications of Differentiation & Anti-Differentiation

STUDENT'S NAME MARKING KEY [KRISZYK]

DATE: Friday 5th March **TIME:** 25 minutes **MARKS**: 27

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

Determine the gradient function of the following. Do not simplify your answers.

(a) $y = x^3(x^4+3x)^7$ [3]

$u = x^3$

$u' = 3x^2$

$v = (x^4+3x)^7$

$v' = 7(x^4+3x)^6(4x^3+3)$

$\frac{dy}{dx} = u'v + v'u$

$= 3x^2(x^4+3x)^7 + 7(x^4+3x)^6(4x^3+3)(x^3)$

(b) $y = \frac{1-x^2}{1+x^2}$ $\begin{matrix} u \\ v \end{matrix}$ [2]

$u' = -2x$

$v' = 2x$

$\frac{dy}{dx} = \frac{u'v - v'u}{v^2}$

$= \frac{-2x(1+x^2) - 2x(1-x^2)}{(1+x^2)^2}$

2. (5 marks)

Determine:

(a) $\int \frac{x^2 - 3x + 1}{x^4} dx$ [3]

$= \int \frac{x^2}{x^4} - \frac{3x}{x^4} + \frac{1}{x^4} dx$

$= \int x^{-2} - 3x^{-3} + x^{-4} dx$

$= \frac{-1}{x} + \frac{3}{2x^2} - \frac{1}{3x^3} + c$

(b) $\int \frac{2}{\sqrt[3]{5x-6}} dx$ [2]

$\frac{2}{5} \int \frac{5}{2} 2(5x-6)^{-1/3} dx$

$= \frac{2}{5} \frac{(5x-6)^{2/3}}{\frac{2}{3}} + c$

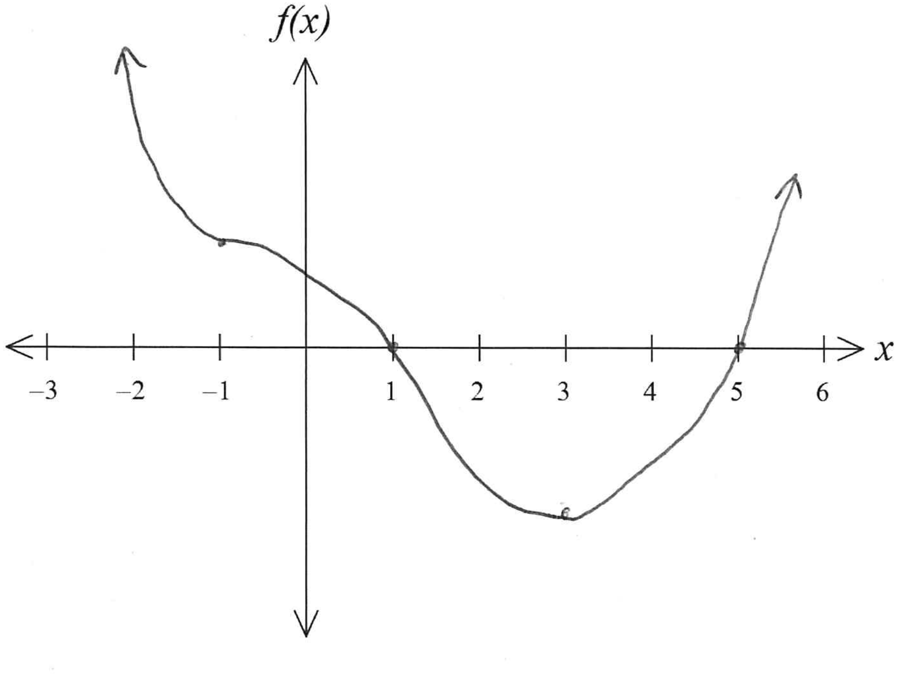
$= \frac{6(5x-6)^{2/3}}{10} + c$

$f(x) = 5x-6$
$f'(x) = 5$

3. (4 marks)

Sketch a possible graph of the function with all the features described below.

- $f(x) = 0$ for $x = 1$ and $x = 5$ — roots @ (1, 0) (5, 0)
- $f'(-1) = f'(3) = 0$ — S.P @ -1 and 3
- $f'(x) \le 0$ for $x < -1$ and $-1 < x < 3$ — f(x) has negative gradient $x < -1$ and between -1 and 3.
- $f''(-1) = 0$, $f''(1) = 0$ — Inflection Pts at -1 and 1



4. (5 marks)

Determine the values of constants p and q given:

$f(x) = px^q$ and $x^2.f''(x) - 2x.f'(x) - 3f(x) = 3x^4$

$f'(x) = qpx^{q-1}$ ✓

$f''(x) = q(q-1)px^{q-2}$ ✓

$x^2\left[q(q-1)px^{q-2}\right] - 2x\left[qpx^{q-1}\right] - 3px^q = 3x^4$

$q(q-1)px^q - 2pqx^q - 3px^q = 3x^4$ ✓

$x^q\left(q(q-1)p - 2pq - 3p\right) = 3x^4$

$\therefore\ x^q = x^4$ $\qquad q(q-1)p - 2pq - 3p = 3$

$\underline{\underline{q = 4}}$ ✓ $\qquad 12p - 8p - 3p = 3$

$\underline{\underline{p = 3}}$ ✓

5. (7 marks)

The curve $y = \frac{x^2}{a+bx}$ has a turning point at $(-1, -0.5)$. If a and b are constants, determine the tangent to the curve at $(1, \frac{1}{6})$.

$$y' = \frac{2x(a+bx) - x^2(b)}{(a+bx)^2} = 0 \checkmark$$

When $x = -1$

$$2(-1)(a-b) - b = 0$$
$$-2a + 2b - b = 0$$
$$b - 2a = 0 \checkmark$$

Sub $(-1, -\frac{1}{2})$ into y.

$$-\frac{1}{2} = \frac{(-1)^2}{a-b}$$
$$a - b = -2$$
$$a = -2 + b \checkmark$$

Solve simultaneously:

$$b - 2a = 0$$
$$b - 2(-2+b) = 0$$
$$b + 4 - 2b = 0$$
$$b = 4 \checkmark$$

$$a = -2 + b$$
$$a = -2 + 4 \checkmark$$
$$a = 2$$

$$\therefore y' = \frac{2x(2+4x) - 4x^2}{(2+4x)^2}$$
$$= \frac{4x + 4x^2}{(2+4x)^2} \checkmark$$

$$m \ @ \ (1, \tfrac{1}{6}) = \frac{4(1) + 4(1)^2}{(2+4(1))^2}$$
$$= \frac{8}{36} \text{ or } \frac{2}{9} \checkmark$$

TRINITY COLLEGE

Year 12 Mathematics Methods Unit 3 & 4
Test 1 2021

Section 2 Calculator Assumed

Applications of Differentiation & Anti-Differentiation

STUDENT'S NAME MARKING KEY [KRISZYK]

DATE: Friday 5th March **TIME:** 25 minutes **MARKS**: 24

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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6. (4 marks)

On analysis of market conditions, a company finds that the profit function for production and sales (in \$) of its line of dishwashers is found to be:

$P(x) = 0.3x^2 - 2x - 500$ for $0 \le x \le 500$ where x is the number of dishwashers produced and sold.

(a) Explain the significance of $P'(\frac{10}{3}) = 0$ [1]

this is when the rate of profit is zero suggesting the rate of profit after this point will be positive. ✓

(b) Determine the marginal profit when 200 dishwashers are produced and sold and explain what this value represents. [3]

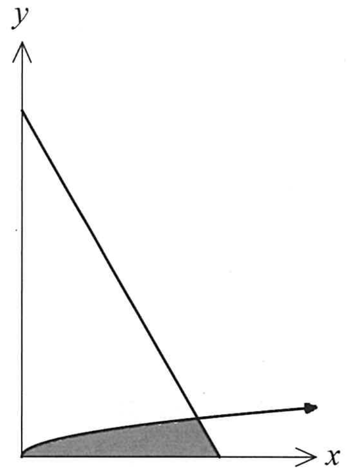
$P'(x) = 0.6x - 2$ ✓

$P'(200) = 118$ ✓

When 200 dishwashers are being produced the profit of producing one more will be \$118. ✓

7. (5 marks)

The diagram below shows the graph of the function $y = \sqrt{x}$ and the straight line $y = -4x + 18$. Determine the shaded area shown in the diagram.



root of line $y = -4x + 18$ occurs at $x = 4.5$ ✓

$y = \sqrt{x}$ and $y = -4x + 18$ intersect at $x = 4$ ✓

$$\therefore \text{Area} = \int_0^4 \sqrt{x}\,dx + \int_4^{4.5} -4x + 18\,dx \quad ✓✓$$

$$= 5.\bar{3} + 0.5$$

$$= 5.8\bar{3} \text{ units}^2 \quad ✓$$

8. (4 marks)

A small metal sphere with a volume of 3.5 cm^3 is dipped in gold. The coating of the gold results in the volume of the sphere increasing to 3.84 cm^3. Use the increments formula to approximate the increase in the radius of the sphere.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2 \checkmark$$

$$\Delta r = \frac{dr}{dV} \times \Delta V$$

$$= \frac{1}{4\pi r^2} \times \Delta V$$

$$= \frac{1}{4\pi r^2} \times 0.34 \checkmark$$

When $V = 3.5$

$$3.5 = \frac{4}{3}\pi r^3$$

$$r = 0.94187 \checkmark$$

$$\therefore \Delta r = \frac{1}{4\pi(0.94187)^2} \times 0.34$$

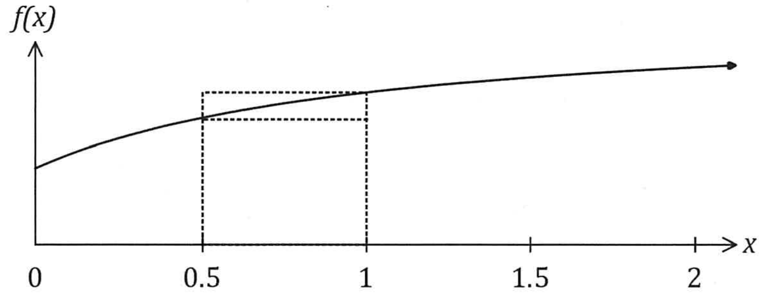
$$\Delta r = 0.03050$$

$\checkmark$

radius increases by approx 0.0305 cm

9. (5 marks)

The graph of $f(x) = \dfrac{6x+2}{x+1}$ is shown below.



Two rectangles are also shown on the graph, with dotted lines, and they both have corners just touching the curve. The smaller is called the inscribed rectangle and the larger is called the circumscribed rectangle.

(a) Complete the missing values in the table below. [1]

x	0	0.5	1	1.5	2
$f(x)$	2	$\frac{10}{3}$	4	$\frac{22}{5}$	$\frac{14}{3}$

✓

(b) Complete the table of areas below and use the values to determine a lower and upper bound for $\int_0^2 f(x)\,dx$. [4]

x interval	0 to 0.5	0.5 to 1	1 to 1.5	1.5 to 2
Area of inscribed rectangle	1	$\frac{5}{3}$	2	$\frac{11}{5}$
Area of circumscribed rectangle	$\frac{5}{3}$	2	$\frac{11}{5}$	$\frac{7}{3}$

✓ ✓

Lower bound : $1 + \frac{5}{3} + 2 + \frac{11}{5} = \frac{103}{15} \sim 6.867$ ✓

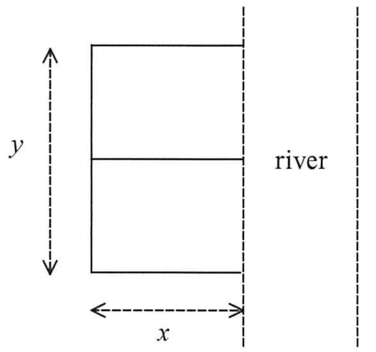
Upper bound : $\frac{5}{3} + 2 + \frac{11}{5} + \frac{7}{3} = \frac{41}{5} \sim 8.2$ ✓

10. (6 marks)

A farmer has \$1500 available to build an E-shaped fence along a straight river so as to create two identical rectangular pastures.

The materials for the side parallel to the river cost \$6 per metre and the materials for the three sides perpendicular to the river costs \$5 per metre.

Each of the sides perpendicular to the river is x metres long, and the side parallel to the river is y metres long.



(a) Assuming that the farmer spends the entire \$1500, show that the total area $A(x)$ of the two pastures, in square metres, is $A(x) = \frac{5}{2}(100x - x^2)$. [2]

$$6y + 5(3x) = 1500 \qquad A = xy$$
$$6y + 15x = 1500 \qquad = x\left(\frac{500-5x}{2}\right)$$
$$2y + 5x = 300$$
$$y = \frac{500-5x}{2} \checkmark \qquad A = \frac{5(100x - x^2)}{2} \checkmark$$

(b) Use calculus techniques to determine the dimensions of the fence that maximises the total area of the pastures and state this area. [4]

$$\frac{dA}{dx} = -5(x-50) \checkmark$$

Solve $\frac{dA}{dx} = 0$ $\checkmark$

$$x = 50$$

$$\left.\frac{d^2A}{dx^2}\right|_{x=50} = -5 \quad \therefore \text{ Maximum T.P.} \checkmark$$

$x = 50\text{ m}$

$y = 125\text{ m}$

Max Area $= 6250\text{ m}^2$ $\checkmark$